Mathematical Modeling and Analysis



Error Rate Fluctuations in High Speed Optical Fiber Communication Systems

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Objective: to verify of general results on statistical properties of error fluctuations in comunication systems with structural disorder and temporal noise.

Today's volume of global information exchange is rapidly growing. Due to huge natural bandwidth, optical fibers appear to be the only media capable of supporting the growth of information that needs to be transmited. In optical fibers information is trismitted digitally. A bit 1 is represented by sending a pulse, and 0 is represented by an absence of a pulse. Real optical fibers are lossy. This requires amplification of the signal about every 100 km. In addition to signal amplification, optical amplifiers spontaneously emit photons, producing short correlated noise. Such noise affects each optical pulse's shape and energy, and eventually might cause bit pattern deterioration.

Birefrengence of optical fiber represents another problem, of quite different nature. It results in the so called polarization mode dispersion (PMD). The cross section of a real fiber is slightly elliptical. Residual ellipticity and ellipse orientation are randomly varying along the fiber. Due to the changing cross-sectional geometry of the fiber core the signal will be coupled into different modes of polarization. This will result in non-deterministic dispersion of the signal.

The distortions of the signal could lead to an improper interpretation of the sent bits, causing errors. If the fiber imperfections do not change in time, the system performance is measured using the so called bit error rate (BER), defined as ratio of erroneous bits to the total bits sent. In reality,

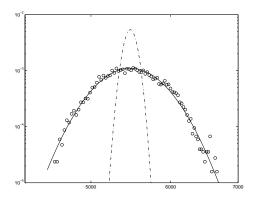


Figure 1. Lognormal fit of PDF of number of errors per measurement. Experimentally obtained histogram (circles). Lognormal fit (solid). Poisson distribution for the same average value of errors (dotted curve).

however, the fiber disorder changes slowly due to the changing external conditions (temperature, stress, etc.) Therefore, a single value of BER is insufficient to describe system performance, and statistical properties of BER fluctuations are required for a complete characterization of system quality.

The probability distribution function (PDF) of BER has lognormal form with long extanded tails [2]:

$$\log(P(B)) \sim -\frac{D_{\xi}^2 Z}{D_m I_0^2} \ln^2\left(\frac{B}{B_0}\right)$$

where D_{ξ} is the strength of amplifier noise, D_m describes the strength of structural disorder, B_0 is the average value of BER, I_0 is the total intensity of the signal, and Z is transmision length.

In spring of 2004 an experiment was set up to verify this result. The experiment was performed in the Optical Sciences Center at the University of Arizona, in the lab of Dr. Franko Kueppers. Numerical and theoretical data analysis was performed during this summer program. In the experiment, BER was measured for a linear, 10 Gbit/s system, with compensated chromatic dispersion. Bits were sent through about 50 km of single mode fiber. The identification of incoming bits were determined by integrating their energy and then comparing that energy to a selected (and fixed) threshhold value I_d . Thus, if the energy of

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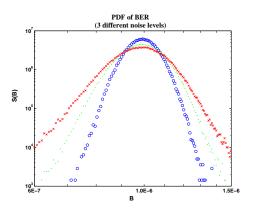


Figure 2. PDF of BER for three values of ASE noise in the system. Here \times has the least noise and o has the most noise in the system. (PDFs were shifted for the purpose of the comparing the tails

the bit is less than I_d the bit is interpreted as 0, otherwise it is interpreted as 1. Local BER was measured each 0.1 seconds (this corresponds to 10^9 bits).

In Figure 1, the histogram (indicated by circles) for the number of errors per BER measurement is presented. On a log-log scale, the histogram strongly resembles a parabola. A parabolic fit is indicated by the solid line. It is possible that these fluctuations of BER are Poisson distributed. The dotted curve represents Poisson distribution for the same average value as the hitogram. Clearly, the distribution that was acquired experimentally is much broader than Poisson distribution and fits the lognormal distribution.

An important observation is that increasing the noise level in the system should cause a distribution of errors to narrow. The noise fluctuations will dominate the PMD fluctuations, and the distribution function will approach the Poisson distribution. In Figure 2, the three curves represent PDFs for the system with three different noise strengths D_{ξ} . Here, for the purpose of comparison, the distributions are shifted to have the same averages. The PDF in circles is for the system with the greatest amount of noise, and in crosses - with the least amount of noise. This corresponds to the behavior predicted in [2].

The importance of this result is due to the fact

that the tails of the distribution are much longer then originally suspected. In the engineering world it is a common practice to assume that the PDF of BER is Gaussian. This work has shown that BER has a lognormal distribution, which is much broader than Gaussian. This means that the probability of a BER much larger then the average value is significant. Moreover, trying to improve the system by supressing the noise levels is not sufficient. It is necessary to compensate for the PMD effects as well.

Conclusion: Bit error rate (BER) is fluctuating in the systems with structural disorder and temporal noise. Its mean value is not sufficient for characterization of system performance. Large error fluctuations are possible due to the extended tails of the probability distribution function of BER. Tail behaviour of PDF is determined by the ratio of the square of temporal noise strength to the strength of structural disorder. Therefore, even for weak noise and disorder, the effect is not necessarily small. Understanding the tail behavior suggests general strategy for improving system performance. Improvement can be achieved by minimizing the mean value of BER and suppressing the tails of the PDF.

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